Nucleosynthesis and the Mass of the τ Neutrino

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Abstract

The primordial abundance of long-lived heavy Majorana neutrinos is calculated from the full Boltzmann equation. Inclusion of scattering reactions drastically change the predicted abundance of a heavy neutrino species. This loosens the well known mass constraint on MeV neutrinos from Big Bang nucleosynthesis, and allows for the existence of a Majorana τ neutrino with mass $m_{\nu_\tau} \geq 11$ MeV. Further experimental efforts are therefore needed to investigate the range $11\text{MeV} \leq m_{\nu_\tau} \leq 24\text{MeV}$. Some interesting cosmological consequences of an MeV ν_τ are also pointed out.

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The possibility of heavy τ neutrinos in the MeV region has been investigated many times in the literature, primarily because such a neutrino could have very interesting consequences for both cosmology and supernovae [1]. The best current experimental limit to the τ neutrino mass comes from the ALEPH collaboration [2], and is $m_{\nu_\tau} \leq 24$ MeV with 95 % CL. It is well known that primordial nucleosynthesis puts stringent limits on the mass and lifetime of the τ neutrino [3–9]. These bounds are most stringent for neutrinos in the MeV region. This is because such a neutrino decouples at a temperature of a few MeV, where its abundance is still comparable to the density of the massless species. But when the temperature drops, the energy density of the massive neutrinos grows relative to that of massless particles. For low temperatures, the energy density of an MeV neutrino will be many times that of the other particles present, effectively making the universe mass dominated, and significantly changing the outcome of nucleosynthesis. If the mass is much higher, the neutrinos will have been Boltzmann suppressed at the time they decouple, to a level where they are not significant during nucleosynthesis.

The most recent investigations of the effects of a massive Majorana τ-neutrino in the MeV region on nucleosynthesis, are those of Kolb et al. [8] and Kawasaki et al. [9]. For a standard Majorana neutrino that is stable on the timescale of nucleosynthesis Kolb et al. find an excluded mass interval of 0.5-32 MeV, whereas Kawasaki et al. find an exclusion interval of 0.1-50 MeV. Combined with the experimental data this means that a massive τ neutrino with mass greater than 0.1-0.5 MeV and lifetime greater than 10^3 s is excluded by nucleosynthesis. However, most of these calculations, except [9], use the integrated Boltzmann equation. Furthermore, they assume that the distribution function is kept in kinetic equilibrium at all times by scattering reactions [10,11]. Kawasaki et al. [9] have used the full equation, but assuming only annihilation interactions, no scattering. Furthermore they use Boltzmann statistics. It is therefore important to see how these limits change when we use the full Boltzmann equation, with all possible interactions. Below we calculate the relic abundance of a massive Majorana τ neutrino with lifetime longer than the time of nucleosynthesis ($\tau \geq 10^3$ s). We then use nucleosynthesis to put limits on the allowed mass

range, and find that a significant window for $m_{\nu_\tau} \geq 11$ MeV is still open.

The fundamental equation used is the Boltzmann equation that describes the evolution of a particle species. We assume that all particle distributions are homogeneous in space and isotropic in phase-space. In that case the Boltzmann equation can be written as

$$C_{\mathrm{L}}[f] = \sum C_{\mathrm{coll}}[f], \tag{1}$$

where $C_{\mathrm{L}}[f] = \partial f/\partial t - Hp\partial f/\partial p$ is the Liouville operator, and $\sum C_{\mathrm{coll}}$ is the sum of all possible collisional interactions. We follow Ref. [12] in assuming that only two-particle interactions are important. The collision operator can then be written as

$$\begin{aligned} C_{\mathrm{coll}}[f] = \frac{1}{2E_1} \int d^3\tilde{p}_2 d^3\tilde{p}_3 d^3\tilde{p}_4 \Lambda(f_1, f_2, f_3, f_4) \times \\ S \mid M \mid^2_{12\to 34} \delta^4(p_1 + p_2 + p_3 - p_4)(2\pi)^4, \end{aligned} \tag{2}$$

where $\Lambda(f_1, f_2, f_3, f_4) = (1 - f_1)(1 - f_2)f_3 f_4 - (1 - f_3)(1 - f_4)f_1 f_2$ is the phase space factor, including Pauli blocking of the final states, and $d^3\tilde{p} = d^3p/((2\pi)^3 2E)$. S is a symmetrization factor of $1/2!$ for each pair of identical particles in initial or final states, and $|M|^2$ is the weak interaction matrix element squared, and appropriately spin summed and averaged. p_i is the four-momentum of particle i. Details of how to solve this equation, as well as the relevant matrixelements, are given in Ref. [12]. In addition to the Boltzmann equation we need equations relating time, temperature and the expansion rate, H. These will be supplied by the energy conservation equation $d(\rho R^3)/dt + pd(R^3)/dt = 0$, and the Friedmann equation $H^2 = 8\pi G\rho/3$. It is assumed, at all times, that $e^\pm$ are kept in thermodynamic equilibrium with the photon gas by electromagnetic interactions. We parametrize the momenta of all particles in terms of the parameter $z = p_\nu/T_\gamma$, and use a grid of 25 points to cover the range in z. The system of equations is then evolved in time by a Runge-Kutta integrator, giving us the distribution function of each particle species, as well as the scale factor R, as a function of photon temperature. By doing it this way, we have the advantage that we know the specific distribution functions for all neutrino species, whereas a treatment using the integrated Boltzmann equation has to make an assumption of kinetic equilibrium,

giving only a rough approximation to the true distribution functions (in fact we find that the distributions can deviate substantially from kinetic equilibrium).

Fig. 1 shows the final relic density of ν_τ before decay becomes important. rm_ν is defined as $rm_\nu = m_\nu n_\nu / n_\nu(m=0)$. We have compared our results to those of Kolb et al. [8], who use the integrated Boltzmann equation. For small masses, it is not important whether we use the integrated or the full Boltzmann equation. The reason is that the neutrino decouples before annihilation really commences, therefore the treatment of the annihilation terms is not very important. For masses larger than 5 MeV there is, however, a significant difference, increasing with mass. This is an effect of large deviations from kinetic equilibrium. The cross-sections increase with energy. Thus the high momentum states are depleted relatively more than low momentum states, only partly compensated by upscattering. For $m_\nu \gg T$ the thermally averaged annihilation cross section, $<\sigma|v|>$, is larger than the corresponding value calculated for a kinetic equilibrium distribution, as normally used in the integrated Boltzmann equation. This is the main reason why the final abundance of ν_τ is lower if we use the true Boltzmann equation rather than the integrated one.

In Fig. 2 we show the final distribution of a 10 MeV ν_τ. The difference between the distributions with and without scattering interactions is striking. If only annihilations are taken into account, the high momentum states are almost empty, because they cannot be refilled. On the other hand, the low momentum states are highly populated, because they annihilate more slowly and cannot be scattered away. The result of this effect is clearly seen in Fig. 1. For large values of m_{ν_τ}, the number density is much higher if we use only annihilation interactions, because once the high momentum states are depleted, almost no annihilation will take place. Fig. 2 also shows the very significant deviation between the actual distribution and a kinetic equilibrium distribution, $f_\nu = 1/(e^{(E-\mu)/T}+1)$, with the same number density.

Fig. 3 shows the final distribution of the electron neutrino. It is seen that the distribution functions can become highly non-thermal, because the pairs created by annihilation of τ neutrinos cannot thermalize sufficiently before the electron neutrinos decouple completely.

This non-thermality can have a significant effect on the production of light elements, especially ^{4}He, because the n-p converting reactions depend not only on the energy density of neutrinos, but also on the specific form of the distribution function for ν_e, which enters in the n-p conversion rate integrals. Overall, the increase in the cosmic expansion rate due to the extra energy density in τ- (and massless) neutrinos, which increases ^{4}He-production, is to some extent compensated by a decrease in neutron-fraction due to the change in the n-p rate integrals.

We also note that the final number densities of muon and electron neutrinos exceed the result for the standard scenario with 3 low-mass flavors by a significant amount. For eV-mass ν_μ or ν_e this changes the present day contribution to the cosmic density to $\Omega_\nu h^2 = \alpha m_\nu/93.03$eV with $\alpha = 1$ for a massless ν_τ, and α =1.07(1.10), 1.07(1.15), 1.09(1.24), 1.16(1.51) for $\nu_e(\nu_\mu)$ for m_{ν_τ} =5, 10, 15, 20 MeV (h is the Hubble-parameter in units of 100km s^{-1} Mpc^{-1}). Later decay of ν_τ can further increase the value of α. This may have significant implications for galaxy formation scenarios.

In order to derive mass limits on ν_τ we need to calculate the predicted abundances of the different light elements and compare them with observations. To do this, we have changed the nucleosynthesis code of Kawano [13] to incorporate a massive Majorana τ neutrino. As previously mentioned, we assume that decay has no effect during nucleosynthesis ($\tau \geq 10^3$s). Note that we not only have to change the energy density of the τ neutrino, but also the energy density in massless neutrinos, as well as the specific distribution function, f_{ν_e}, of electron neutrinos in the nucleosynthesis code.

Fig. 4 shows the relic abundances of the different light elements as a function of the baryon-to-photon ratio, η, for different m_{ν_τ}.

The observational limits on the primordial ^{4}He abundance have been evaluated by Olive and Steigman [14] to be $Y_P = 0.232 \pm 0.003 \pm 0.005$, where the first uncertainty is a 1 σ statistical uncertainty, and the second an estimated 1 σ systematic uncertainty. However, the actual systematic uncertainty might be substantially larger than this. Copi et al. [15] quote a systematic uncertainty of $+0.016/-0.012$ as more realistic. Furthermore there

are some indications that the ^{4}He abundance has been systematically underestimated, and might really be as large as 0.255 [16].

A lower limit to the primordial deuterium abundance has been calculated by Hata et al. [17], from interstellar medium data, to be $D/H \geq 1.6 \times 10^{-5}$. The recent observation of an apparently very high deuterium abundance in QSO absorption systems would imply an upper limit of around 2×10^{-4} [18–20]. The interpretation of these observations is, however, uncertain [21]. Normally, one uses an upper limit on $D +^3$ He instead of D alone. This gives $(D +^3 \mathrm{He})/H \leq 1.1 \times 10^{-4}$ [15] (again from local galactic data), incompatible with the high deuterium value. The upper limit from this method is unfortunately also very uncertain, because it depends on evolution effects of ^{3}He, which are not particularly well known [20]. Finally the abundance of ^{7}Li is also used. Copi et al. [15] use a limit of $^7\mathrm{Li}/H = 1.4 \pm 0.3^{+1.8}_{-0.4} \times 10^{-10}$.

Based on these data we use a strong limit to the primordial abundances of $Y_P = 0.232 \pm 0.006$, $D/H \geq 1.6 \times 10^{-5}$, $(D+^3\mathrm{He})/H \leq 1.1 \times 10^{-4}$ and $^7\mathrm{Li}/H = 1.4^{+1.8}_{-0.5} \times 10^{-10}$. Furthermore we use a weak (and perhaps rather more realistic) limit of $Y_P = 0.232^{+0.016}_{-0.012}$, $D/H \geq 1.6 \times 10^{-5}$, $D/H \leq 2 \times 10^{-4}$ and $^7\mathrm{Li}/H = 1.4^{+1.8}_{-0.5} \times 10^{-10}$.

We now use these limits to infer mass limits on the τ neutrino. Using the strong limit we obtain a minimum allowed mass of 20 MeV, and using the weak limit we obtain a minimum mass of 11 MeV. Thus, in contrast to the conclusion of earlier work, there is still an allowed mass interval of 11–20 MeV $\leq m_{\nu_\tau} \leq$ 24 MeV [22]. These limits all apply to Majorana neutrinos. Similar effects will occur for Dirac neutrinos, also opening a new mass window for them, but the calculations with the full Boltzmann equations are complicated by the need to include spin-flip interactions.

Our lower limits on the τ neutrino mass are much weaker than those of Kolb et al. and Kawasaki et al. As discussed previously, this is to be expected since Kolb et al. use the integrated Boltzmann equation and Kawasaki et al. only consider annihilation interactions. Thus the conclusions of earlier works, that an MeV τ neutrino is excluded by nucleosynthesis, is changed if we use the full Boltzmann equation with all possible interactions. New

experiments to directly search for a neutrino in this mass range are therefore needed. Furthermore, if the true ^{4}He abundance turns out to be high, as pointed to by some authors [16], and the baryon density low, as suggested by the lack of microlensing events in the galactic halo, a heavy τ neutrino could be a natural way to bring the predictions of Big Bang nucleosynthesis into consistency with observations.

ACKNOWLEDGMENTS

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[22] Another standard manner of expressing nucleosynthesis results is in terms of the equivalent number of massless neutrinos, N_ν, because the ^{4}He-production is roughly given by $Y_P \approx 0.2253 + 0.011 \ln \eta_{10} + 0.0139(N_\nu - 3)$. We find that N_ν takes the values 3.80, 3.47, 2.90, and 2.00 for m_{ν_τ} of 10, 15, 20 MeV and ∞. Using $N_\nu \leq 3.4$ Kolb et al. [8] obtain a bound of 25 MeV (above the current experimental limit), which for our results is reduced to 16 MeV (well below the experimental limit).

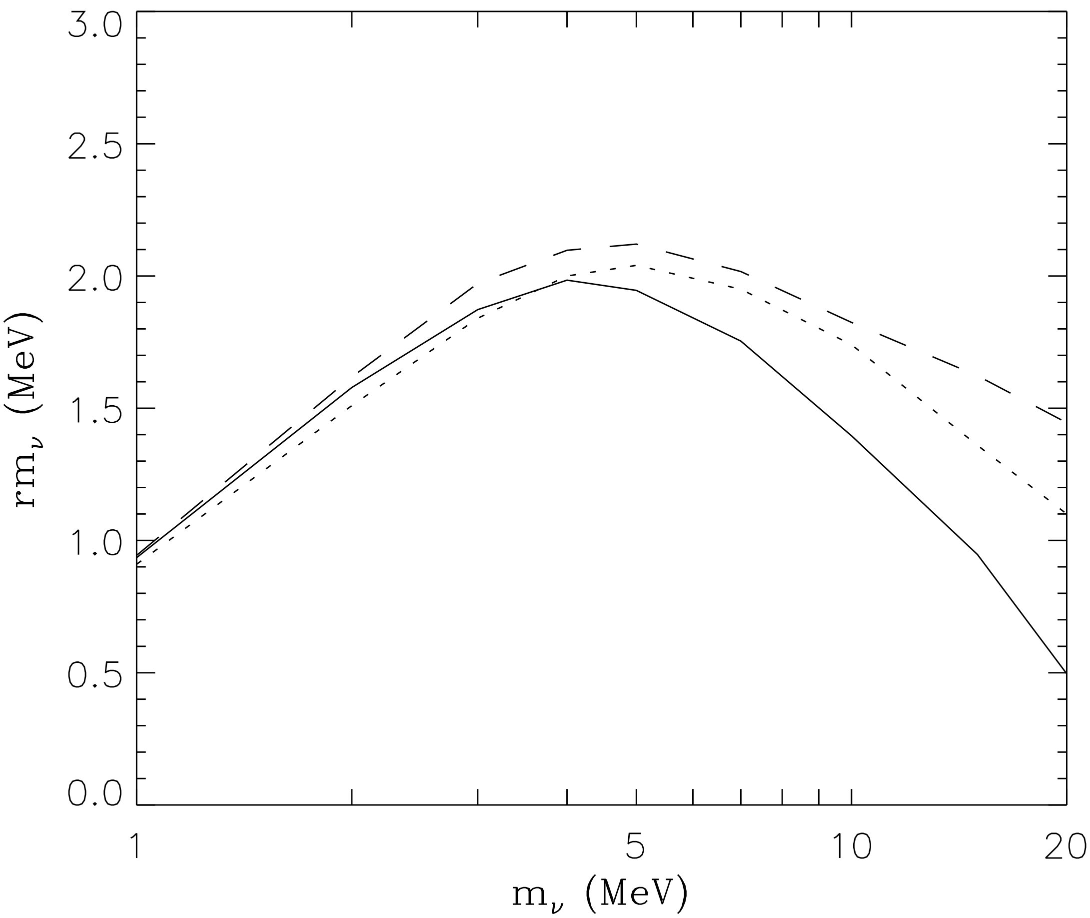
FIGURES

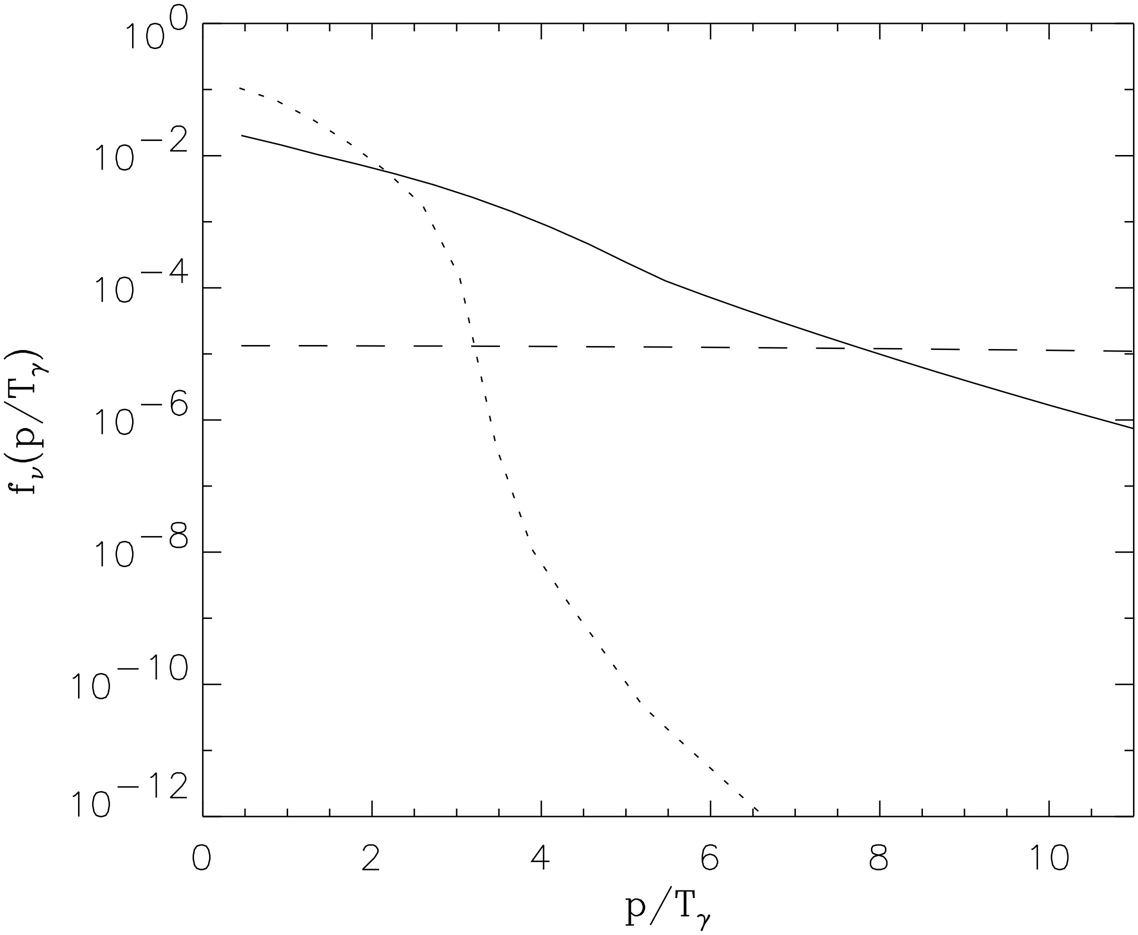
FIG. 1. The relic number density times mass, rm_ν, for massive τ neutrinos. The solid curve is calculated using the full Boltzmann equation and all interactions. The dashed curve is obtained by using the full equation, but with only annihilation interactions. The dotted curve is adopted from Kolb et al. [8]

FIG. 2. The distribution of τ neutrinos of mass 10 MeV at a photon temperature of 0.03 MeV. The solid line comes from using all possible interactions and the dotted curve is from using only annihilations. The dashed curve is a kinetic equilibrium distribution with the same number density as that of the solid curve. Notice that the actual distribution is far from kinetic equilibrium.

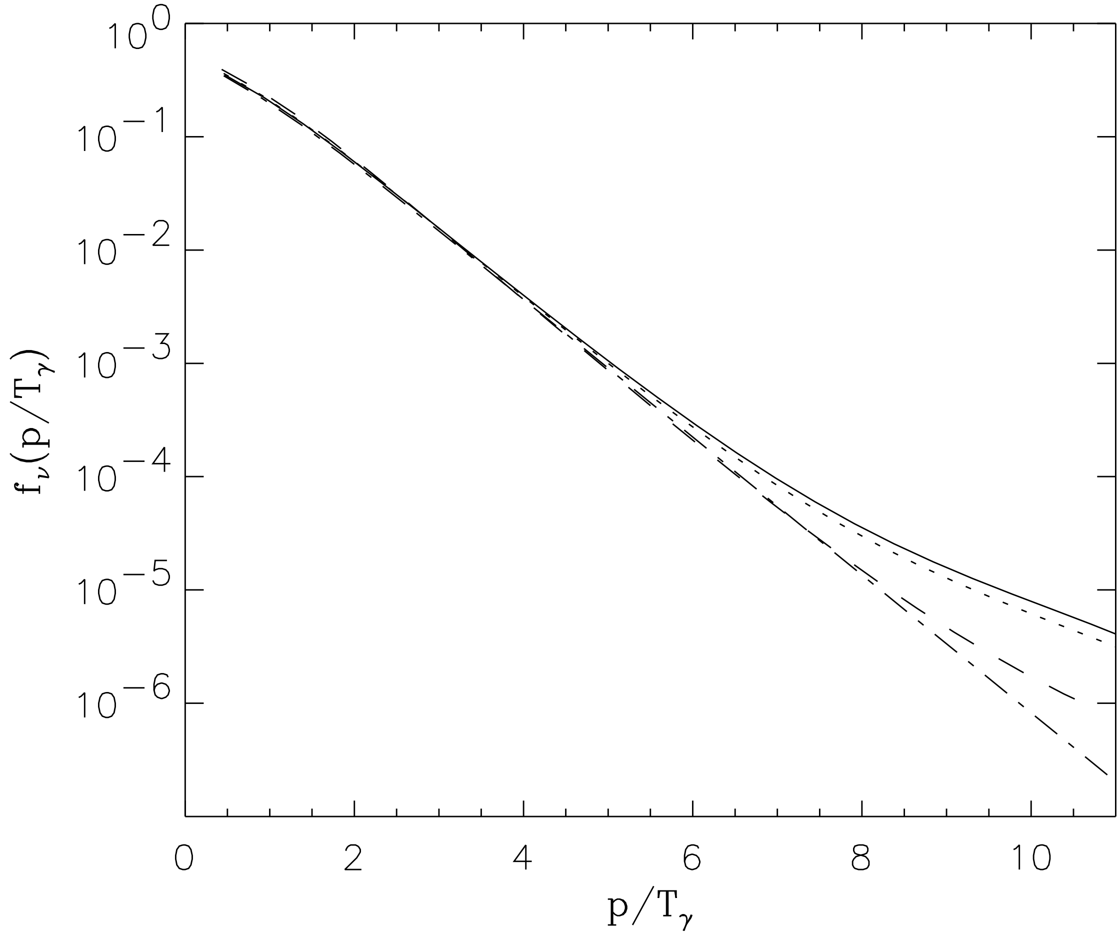
FIG. 3. The electron neutrino distribution at a photon temperature of 0.03 MeV. The solid curve is for $m_{\nu_\tau} = 5$ MeV, the dotted for $m_{\nu_\tau} = 10$ MeV and the dashed for $m_{\nu_\tau} = 15$ MeV. For comparison, a standard ν_e-distribution without input from ν_τ-annihilations is shown as the dot-dashed curve.

FIG. 4. Predicted light element abundances as a function of the baryon-to-photon ratio $\eta_{10} = 10^{10} n_B/n_\gamma$ for different τ neutrino masses. The solid curves are for $m_{\nu_\tau} = 10$ MeV, the dotted for $m_{\nu_\tau} = 15$ MeV and the dashed for $m_{\nu_\tau} = 20$ MeV. The dot-dashed curves are the predictions for $m_{\nu_\tau} \to \infty$, corresponding to 2 massless neutrinos. Horizontal lines are observational limits, the dot-dot-dot-dashed lines correspond to the strong limit and the long-dashed to the weak limit. Note that in some cases the strong and weak limits coincide.


3.0
2.5
2.0
1.5
1.0
0.5
0.0
1
5
10
20
rm_ν (MeV)
m_ν (MeV)



10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
$f_\nu(p/T_\gamma)$
0
2
4
6
8
10
p/T_γ



10^0
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10^{-2}
10^{-3}
10^{-4}
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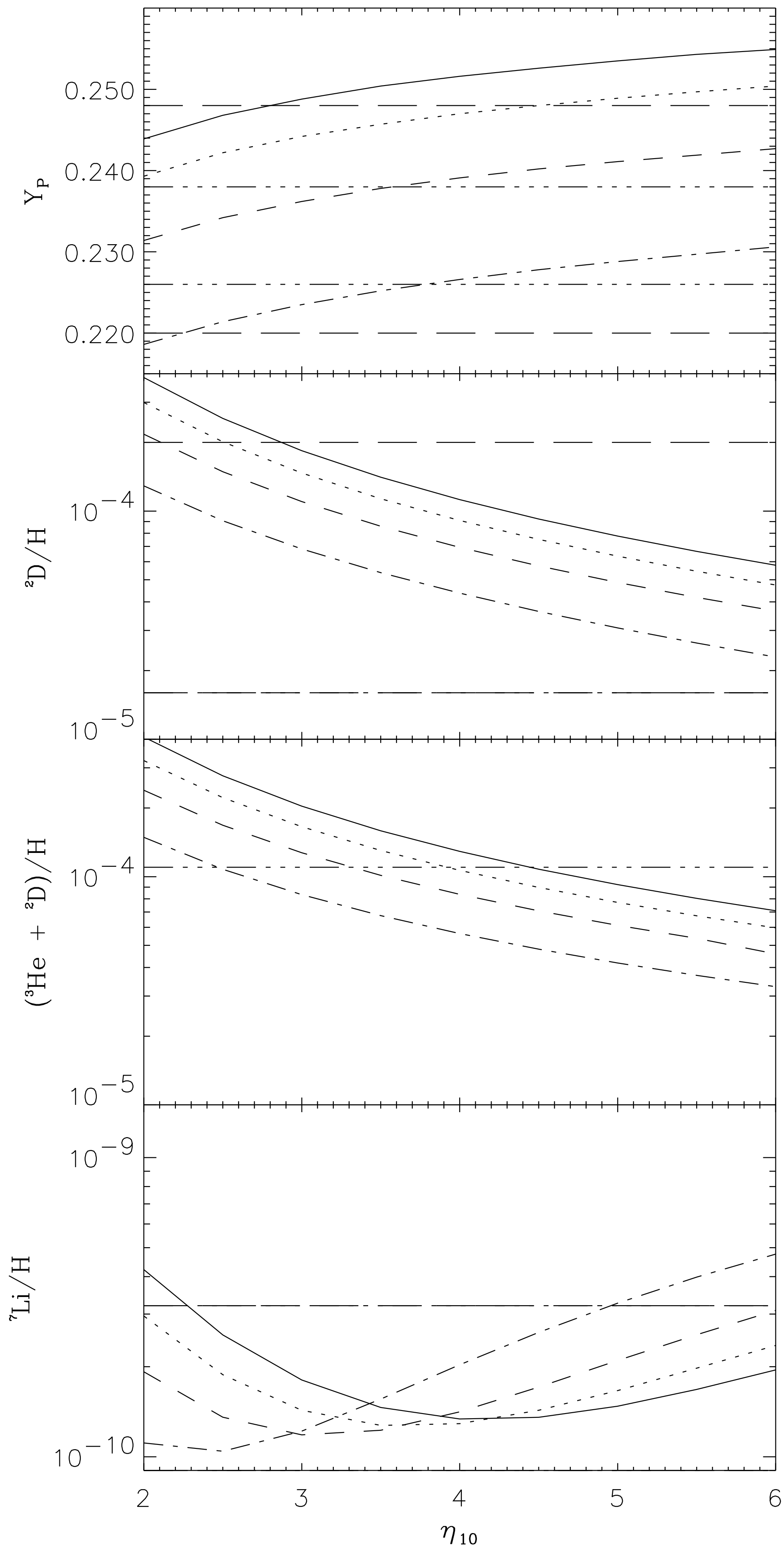
10^{-6}
$f_\nu(p/T_\gamma)$
0
2
4
6
8
10
p/T_γ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0.230
0.220
$^2D/H$
10^{-4}
10^{-5}
$(^3He + {}^2D)/H$
10^{-4}
10^{-5}
$^7Li/H$
10^{-9}
10^{-10}
2
3
4
5
6
η_{10}